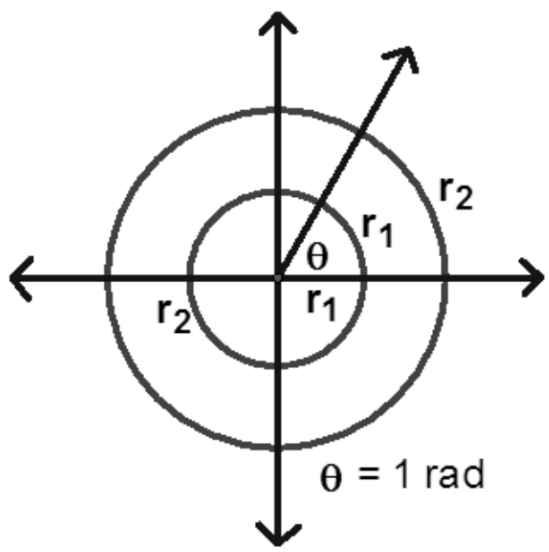
3.2 Angle Measure in Degrees and Radians

A **central angle** of a circle is an angle whose vertex is the center of the circle. When a central angle θ intercepts an arc that has the same length as the radius of the circle, the measure of that angle is one **radian**, abbreviated 1 rad.



To convert from degrees to radians:

Multiply the number of degrees by $\frac{\pi}{180^\circ}$.

To convert from radians to degrees:

Multiply the number of radians by $\frac{180^\circ}{\pi}$.

<u>Example 1</u>: Express each angle measure in radians. Give answers in terms of π.

a. 135°

$135^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{3\pi}{4}$

b. -90°

$-90^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{\pi}{2}$

c. 60°

$60^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3}$

d. -150°

$-150^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{5\pi}{6}$

Example 2: Express each angle measure in degrees. If necessary, round to the nearest tenth of a degree.

a. $\frac{-8\pi}{3}$

$-\frac{8\cancel{\pi}}{3}\left(\frac{180^\circ}{\cancel{\pi}}\right) = -480^\circ$

b. $\frac{8}{3}$

$\frac{8}{3}\left(\frac{180^\circ}{\pi}\right) \approx 152.8^\circ$

c. $\frac{13\pi}{12}$

$\frac{13\pi}{12}\left(\frac{180^\circ}{\pi}\right) = 195^\circ$

d. 5.1

$5.1\left(\frac{180^\circ}{\pi}\right) \approx 292.2^\circ$

The length s of the arc of a circle of radius r determined by central angle θ expressed in radians is given by $s = r\theta$.

<u>Example 3</u>: Find each arc length to the nearest tenth of a centimeter.

$24.5 = \frac{49}{2} \cdot \frac{\pi}{4}$

a. $r = 24.5$ cm $\theta = 45°$

$\theta = 45° = \frac{\pi}{4}$

$s = r\theta$

$s = 24.5\left(\frac{\pi}{4}\right)$

$s = \frac{49\pi}{8}$ cm

$\boxed{s \approx 19.2 \text{ cm}}$

b. r = 4.5 cm $\theta = 0.6\pi$

$$s = r\theta$$

$$s = 4.5\left(\frac{3\pi}{5}\right)$$

$$s = \frac{27\pi}{10}\text{ cm}$$

$$\boxed{s \approx 8.5\text{ cm}}$$

c. r = 7 cm $\theta = 85°$

$$\theta = 85°\left(\frac{\pi}{180}\right) = \frac{17\pi}{36}$$

$$s = r\theta$$

$$s = 7\left(\frac{17\pi}{36}\right)$$

$$s = \frac{119\pi}{36}\text{ cm}$$

$$\boxed{s \approx 10.4\text{cm}}$$

Example 4:

a. A pendulum swings through an angle of 60°, describing an arc 5.0 m long. Determine the length of the pendulum to the nearest tenth of a meter.

θ s r

$\theta = 60° = \frac{\pi}{3}$

$s = r\theta$

$\frac{5}{\frac{\pi}{3}} = \frac{r\left(\frac{\pi}{3}\right)}{\frac{\pi}{3}}$

$\frac{15}{\pi} = r$

$\frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{d}{c}$

$r \approx 4.8$ m

b. A pendulum swings through an angle of $\overset{\theta}{\frac{\pi}{4}}$ rad, describing an arc $\overset{s}{0.4}$ m long. Determine the $\overset{r}{\text{length}}$ of the pendulum to the nearest tenth of a meter.

$$s = r\theta$$

$$\frac{0.4}{\frac{\pi}{4}} = \frac{r\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}}$$

$$\frac{1.6}{\pi} = r$$

$$\boxed{r \approx 0.5\text{ m}}$$

c. A pendulum swings through an angle of 30°, describing an arc 1.2 m long. Determine the length of the pendulum to the nearest tenth of a meter.

$\theta = 30° = \frac{\pi}{6}$

$S = r\theta$

$\frac{1.2}{\frac{\pi}{6}} = \frac{r\left(\frac{\pi}{6}\right)}{\frac{\pi}{6}}$

$\frac{7.2}{\pi} = r$

$\boxed{r \approx 2.3\,m}$

A sector of a circle is a region bounded by a central angle and an intercepted arc. If the radian measure of the central angle is θ then

$$\frac{Area\ Of\ The\ Sector}{Area\ Of\ The\ Circle} = \frac{A_S}{A_{\odot}} = \frac{\theta}{2\pi} \Rightarrow \frac{A_S}{\pi r^2} = \frac{\theta}{2\pi} \Rightarrow \boxed{A_S = \frac{1}{2} r^2 \theta}$$

<u>Example 5</u>: Determine to the <u>nearest tenth</u> of a square unit the area of each sector.

a. circle with radius 9.0 cm intercepted by central angle of 120°.

$\theta = 120° = \frac{2\pi}{3}$

$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2}(9)^2\left(\frac{2\pi}{3}\right)$

$A = \frac{1}{2}(81)\left(\frac{2\pi}{3}\right)$

$A = 27\pi$

$\boxed{A \approx 84.8\ cm^2}$

b. circle with radius 1.6 m intercepted by central angle of $\frac{\pi}{4}$.

$$A = \frac{1}{2} r^2 \theta$$
$$A = \frac{1}{2}(1.6)^2\left(\frac{\pi}{4}\right)$$
$$A = \frac{1}{2}(2.56)\left(\frac{\pi}{4}\right)$$
$$A = \frac{8\pi}{25}$$
$$\boxed{A \approx 1.0\ m^2}$$

c. circle with radius 3.8 in intercepted by central angle of 42°.

$$\theta = 42^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{30}$$

$$A = \frac{1}{2} r^2 \theta$$
$$A = \frac{1}{2}(3.8)^2\left(\frac{7\pi}{30}\right)$$
$$A = \frac{1}{2}(14.44)\left(\frac{7\pi}{30}\right)$$
$$A = \frac{2527\pi}{1500}$$
$$\boxed{A \approx 5.3\ in^2}$$

Homework: pp. 129 - 130 Class Exercises 1 - 12; Special Angle Chart
pp. 130 - 131 Practice Exercises 1 – 20; 23 – 30; 33 – 36