3.2 Angle Measure in Degrees and Radians

A central angle of a circle is an angle whose vertex is the center of the circle. When a central angle $\theta$ intercepts an arc that has the same length as the radius of the circle, the measure of that angle is one radian, abbreviated 1 rad.
To convert from degrees to radians:
   Multiply the number of degrees by $\frac{\pi}{180^\circ}$.

To convert from radians to degrees:
   Multiply the number of radians by $\frac{180^\circ}{\pi}$.
Example 1: Express each angle measure in radians. Give answers in terms of \( \pi \).

a. 135°

\[
135° \left( \frac{\pi}{180°} \right) = \frac{3\pi}{4}
\]

b. -90°

\[
-90° \left( \frac{\pi}{180°} \right) = -\frac{\pi}{2}
\]

c. 60°

\[
60° \left( \frac{\pi}{180°} \right) = \frac{\pi}{3}
\]

d. -150°

\[
-150° \left( \frac{\pi}{180°} \right) = -\frac{5\pi}{6}
\]
**Example 2**: Express each angle measure in degrees. If necessary, round to the nearest tenth of a degree.

a. \( \frac{-8\pi}{3} \)

\[ -\frac{8\pi}{3} \left( \frac{180^\circ}{\pi} \right) = -480^\circ \]

b. \( \frac{8}{3} \)

\[ \frac{8}{3} \left( \frac{180^\circ}{\pi} \right) \approx 152.8^\circ \]

c. \( \frac{13\pi}{12} \)

\[ \frac{13\pi}{12} \left( \frac{180^\circ}{\pi} \right) = 195^\circ \]

d. 5.1

\[ 5.1 \left( \frac{180^\circ}{\pi} \right) \approx 292.2^\circ \]
The length $s$ of the arc of a circle of radius $r$ determined by central angle $\theta$ expressed in radians is given by $s = r\theta$.

**Example 3:** Find each arc length to the nearest tenth of a centimeter.

a. $r = 24.5$ cm $\theta = 45^\circ$

\[
\begin{align*}
\theta &= 45^\circ = \frac{\pi}{4} \\
S &= r\theta \\
S &= 24.5 \left( \frac{\pi}{4} \right) \\
S &= \frac{49\pi}{8} \text{ cm} \\
S &\approx 19.2 \text{ cm}
\end{align*}
\]
b. \( r = 4.5 \text{ cm} \ \theta = 0.6\pi \)

\[
S = r\theta \\
S = 4.5 \left( \frac{3\pi}{5} \right) \\
S = \frac{27\pi}{10} \text{ cm} \\
S \approx 8.5 \text{ cm}
\]

c. \( r = 7 \text{ cm} \ \theta = 85^\circ \)

\[
\theta = 85^\circ \left( \frac{\pi}{180} \right) = \frac{17\pi}{36} \\
S = r\theta \\
S = 7 \left( \frac{17\pi}{36} \right) \\
S = \frac{119\pi}{36} \text{ cm} \\
S \approx 10.4 \text{ cm}
\]
Example 4:
a. A pendulum swings through an angle of $60^\circ$, describing an arc $5.0 \text{ m}$ long. Determine the length of the pendulum to the nearest tenth of a meter.

\[
\Theta = 60^\circ = \frac{\pi}{3}
\]

\[
S = r \Theta
\]

\[
\frac{5}{\frac{\pi}{3}} = r \left( \frac{\pi}{3} \right)
\]

\[
\frac{15}{\pi} = r
\]

\[
r \approx 4.8 \text{ m}
\]
b. A pendulum swings through an angle of $\frac{\pi}{4}$ rad, describing an arc 0.4 m long. Determine the length of the pendulum to the nearest tenth of a meter.

\[
S = r \theta \\
0.4 = r \left(\frac{\pi}{4}\right) \\
\frac{0.4 \pi}{4} = r \\
1.4 \pi = r \\
\]

$r \approx 0.5$ m
c. A pendulum swings through an angle of $30^\circ$, describing an arc $1.2\text{ m}$ long. Determine the length of the pendulum to the nearest tenth of a meter.

$$\Delta - 30^\circ = \frac{\pi}{6}$$

$$s = r\theta$$

$$1.2 = r \left( \frac{\pi}{6} \right)$$

$$\frac{7.2}{\pi} = r$$

$$r \approx 2.3 \text{ m}$$
A sector of a circle is a region bounded by a central angle and an intercepted arc. If the radian measure of the central angle is $\theta$ then

$$\frac{\text{Area Of The Sector}}{\text{Area Of The Circle}} = \frac{A_S}{A_\odot} = \frac{\theta}{2\pi} \implies \frac{A_S}{\pi r^2} = \frac{\theta}{2\pi} \implies A_S = \frac{1}{2} r^2 \theta$$
**Example 5:** Determine to the nearest tenth of a square unit the area of each sector.

a. circle with radius 9.0 cm intercepted by central angle of 120°.

\[ \theta = 120° = \frac{2\pi}{3} \]

\[ A = \frac{1}{2} r^2 \theta \]

\[ A = \frac{1}{2} (9)^2 \left( \frac{2\pi}{3} \right) \]

\[ A = \frac{1}{2} (81) \left( \frac{2\pi}{3} \right) \]

\[ A = 27\pi \]

\[ A \approx 84.8 \text{ cm}^2 \]
b. circle with radius 1.6 m intercepted by central angle of $\frac{\pi}{4}$.

\[ A = \frac{1}{2} r^2 \theta \]
\[ A = \frac{1}{2} (1.6)^2 \left( \frac{\pi}{4} \right) \]
\[ A = \frac{1}{2} (2.56) \left( \frac{\pi}{4} \right) \]
\[ A = \frac{8\pi}{25} \]
\[ A \approx 1.0 \text{ m}^2 \]

c. circle with radius 3.8 in intercepted by central angle of 42°.

\[ \theta = 42^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{7\pi}{30} \]
\[ A = \frac{1}{2} r^2 \theta \]
\[ A = \frac{1}{2} (3.8)^2 \left( \frac{7\pi}{30} \right) \]
\[ A = \frac{1}{2} (14.44) \left( \frac{7\pi}{30} \right) \]
\[ A = \frac{2.527\pi}{150} \]
\[ A \approx 5.3 \text{ in}^2 \]
Homework: pp. 129 - 130  Class Exercises 1 - 12; Special Angle Chart
pp. 130 - 131  Practice Exercises 1 – 20; 23 – 30; 33 – 36