3.3 Angular and Linear Velocity

The formula $s = r\theta$ can also be used to analyze the motion of a point along a circular path. Circular objects that turn about axes through their centers, display **rotary motion**. Consider point P on the edge of a wheel with center O. As the wheel rotates, $\overrightarrow{OP}$ moves through an angle called the **angular displacement** θ of P.

Example 1:

a. A wheel makes $1\frac{1}{4}$ rotations about its axis. Find the angular displacement, in radians, of a point P on the wheel.

$$\theta = \frac{5}{4}(2\pi) = \frac{5\pi}{2}$$

b. A gear makes 1.5 rotations about its axis. What is the angular displacement, in radians, of a point on the gear?

$\theta = \frac{3}{2}(2\pi) = 3\pi$

c. A circular knob is used to advance the paper in a typewriter. The knob makes 3.3 rotations about its axis. Find the angular displacement, in radians, of a point on the edge of the knob.

$$\theta = \frac{33}{10}(2\pi) = \frac{33\pi}{5}$$

The **angular velocity** ω of a point moving in a circular path is the angular displacement of the point per unit of time t, thus

$$\omega = \frac{\theta}{t}$$

Example 2:

a. Determine the angular velocity of the tip of the second hand of a clock in radians per second.

$$\omega = \frac{\theta}{t} = \frac{2\pi}{60} = \boxed{\frac{\pi}{30} \text{ rad/sec}}$$

$$\approx 0.1047 \text{ rad/sec}$$

b. Find the angular velocity of the tip of the minute hand in radians per second.

$$\omega = \frac{\theta}{t} = \frac{2\pi}{3600} = \boxed{\frac{\pi}{1800} \text{ rad/sec}}$$

$$\approx 0.0017 \text{ rad/sec}$$

c. Find the angular velocity of the tip of the hour hand in radians per minute.

$$\omega = \frac{\theta}{t} = \frac{2\pi}{720} = \boxed{\frac{\pi}{360} \text{ rad/min}}$$

$$\approx 0.0087 \text{ rad/min}$$

Example 3:

a. A wheel turns at a rate of 600 rpm. What is the angular velocity of the wheel in radians per second?

$$\frac{600\ \cancel{\text{rev}}}{1\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ \text{sec}} \cdot \frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}} = \frac{1200\pi}{60} = \boxed{20\pi\ \text{rad}/\text{sec}}$$

b. Find the angular velocity in radians per second of a point on a gear turning at the rate of 3.4 rpm.

$$\frac{3.4 \cancel{\text{rev}}}{1 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}} = \frac{17\pi}{150} \text{ rad/sec}$$

c. What is the angular velocity in radians per second of a notch on a wheel turning at a rate of 7600 rpm?

$$\frac{7600 \text{ rev}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{\frac{760\pi}{3} \text{ rad/sec}}$$

An object moves along a circle of radius r at a constant rate, its **linear velocity** V is the distance s traveled along the circumference of the circle per unit of time t. Therefore,

$$V = \frac{s}{t} = \frac{r\theta}{t}$$

where θ is the angular displacement of the object in radians.

Example 4:

a. A second hand of a clock is 8.0 cm long. What is the linear velocity of the tip of this hand?

$$V = \frac{r\theta}{t}$$

$$V = \frac{8(2\pi)}{60}$$

$$\boxed{V = \frac{4\pi}{15} \text{ cm/sec}}$$

$$\boxed{V \approx 0.8378 \text{ cm/sec}}$$

b. The second hand of a watch is 1.1 cm long. What is the linear velocity of the tip of this hand?

$$V = \frac{r\theta}{t}$$

$$V = \frac{1.1(2\pi)}{60}$$

$$\boxed{V = \frac{11\pi}{300} \text{ cm/sec}}$$

$$\boxed{V \approx 0.1152 \text{ cm/sec}}$$

c. A skater moves around the edge of a circular practice rink at the rate of 2 rpm. The rink has a radius of 4.1 m. What is the skater's velocity in meters per minute?

$$V = \frac{r\theta}{t}$$

$$V = \frac{4.1(4\pi)}{1}$$

$$\boxed{V = \frac{82\pi}{5} \text{ m/min}}$$

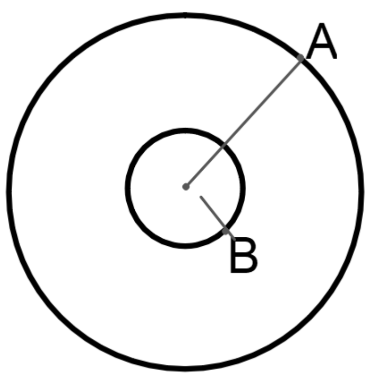
$$\boxed{V \approx 51.5221 \text{ m/min}}$$

If a point travels in a circular path with radius r at a constant speed, linear velocity V, angular velocity ω, and angular displacement θ (measured in radians), then

$$V = \frac{r\theta}{t} = r\frac{\theta}{t} = r\omega$$

Example 5:

a. If the inner radius of a record is 0.5 cm, the outer radius is 15.0 cm, and it rotates at $33\frac{1}{3}$ rpm, compare the linear velocities of the points A and B.



$$\omega = \frac{\frac{100}{3}\text{ rev}}{1\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = \frac{10\pi}{9}\text{ rad/sec}$$

A: $V = r\omega$

$V = 15\left(\frac{10\pi}{9}\right)$

$\boxed{V = \frac{50\pi}{3}\text{ cm/sec}}$

$\boxed{V \approx 52.3599\text{ cm/sec}}$

B: $V = r\omega$

$V = \frac{1}{2}\left(\frac{10\pi}{9}\right)$

$\boxed{V = \frac{5\pi}{9}\text{ cm/sec}}$

$\boxed{V \approx 1.7453\text{ cm/sec}}$

b. A gear rotates at 5 rps. Compare the linear velocities of two points 0.8 cm and 5.2 cm from the center of the gear.

$$\omega = \frac{5 \text{ rev}}{1 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 10\pi \text{ rad/sec}$$

A: $V = r\omega$

$V = \frac{4}{5}(10\pi)$

$\boxed{V = 8\pi \text{ cm/sec}}$

$\boxed{V \approx 25.1327 \text{ cm/sec}}$

B: $V = r\omega$

$V = \frac{26}{5}(10\pi)$

$\boxed{V = 52\pi \text{ cm/sec}}$

$\boxed{V \approx 163.3628 \text{ cm/sec}}$

c. A merry-go-round at a playground spins at 1 revolution per 36 seconds. Compare the linear velocities of two children who are A 2 ft and B 4 ft from the center of the merry-go-round.

$\omega = \frac{\theta}{t} = \frac{2\pi}{36} = \frac{\pi}{18}$ rad/sec

A: $V = r\omega$

$V = 2\left(\frac{\pi}{18}\right)$

$\boxed{V = \frac{\pi}{9} \text{ ft/sec}}$

$\boxed{V \approx 0.3491 \text{ ft/sec}}$

B: $V = r\omega$

$V = 4\left(\frac{\pi}{18}\right)$

$\boxed{V = \frac{2\pi}{9} \text{ ft/sec}}$

$\boxed{V \approx 0.6981 \text{ ft/sec}}$

Example 6:

a. Hans rides a vehicle with large tires of radius 16 in at 24 mph. Find the angular velocity of a tire in radians per min. How many revolutions per minute does the tire make?

$$V = \frac{24 \text{ mi}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = 25{,}344 \text{ in/min}$$

$$V = r\omega$$

$$\omega = \frac{V}{r}$$

$$\omega = \frac{25{,}344}{16}$$

$$\boxed{\omega = 1584 \text{ rad/min}}$$

$$\frac{1584 \text{ rad}}{1 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} = \frac{792}{\pi} \text{ rpm}$$

$$\approx 252.101$$

$$\boxed{\text{about } 252 \text{ rpm}}$$

b. A unicycle as a tire with radius 10 in. It is traveling at a speed of 5.5 miles per hour. Find the angular velocity of the tire in radians per second. How many revolutions per second does the tire make?

$$V = \frac{5.5\text{ mi}}{1\text{ hr}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{12\text{ in}}{1\text{ ft}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{484}{5} = 96.8\text{ in/sec}$$

$$\omega = \frac{V}{r}$$

$$\omega = \frac{96.8}{10}$$

$$\boxed{\omega = 9.68\text{ rad/sec}}$$

$$\frac{9.68}{2\pi} \approx 1.5406$$

$$\boxed{\text{about } 1.5\text{ rps}}$$

Homework: pp. 137 – 138 => Class Exercises 1 – 9;

Practice Exercises 1 - 17